

Fluid Flow on a Rotating Sphere

Centrifugal force:

Due to centrifugal acceleration from its rotation, the earth is slightly oblate rather than perfectly spherical. The distance from the center of the earth to the poles is smaller than the distance from the center of the earth to the equator. Compared to a perfect sphere, the surface of the earth slopes slightly downward from the equator to the poles. Unlike the case for a perfect sphere, the radial acceleration of gravity toward the center of the earth is not exactly perpendicular to the local horizontal surface (except at the poles and the equator). This means there will be a small horizontal component of gravitational acceleration along the surface of the earth and toward the pole. For fluid parcels at rest with respect to the earth, this gravitational acceleration is balanced by an equal and opposite component of centrifugal acceleration along the surface toward the equator. A tabletop rotating parabolic bowl provides a simple analogy to this balance. Gravity accelerates objects downward toward the center of the bowl, corresponding to the pole, and the centrifugal force from rotation accelerates objects upward toward the rim of the bowl, corresponding to the equator.

Coriolis force from zonal (east-west) velocity:

Because the earth's rotation is from west to the east, a fluid parcel with an eastward velocity relative to the earth is rotating faster than the earth in the absolute reference frame. This means the centrifugal acceleration toward the equator is stronger than the gravitational acceleration toward the pole; thus the fluid parcel experiences a net equatorward acceleration. Conversely, a fluid parcel with a westward velocity relative to the earth is rotating more slowly than the earth and experiences a net poleward acceleration because the gravitational force is greater than the centrifugal force. In a rotating reference frame, these effects appear as a Coriolis force.

Coriolis force from meridional (north-south) velocity:

Fluid parcels with meridional velocity conserve angular momentum because they do not experience any torque in the zonal direction. This requires that the rotation rate in the absolute reference frame increase when a fluid parcel moves toward the pole (thus decreasing its distance from the axis of rotation) and decrease when a fluid parcel moves toward the equator (thus increasing its distance from the axis of rotation). An increase in the rate of rotation in the absolute reference frame corresponds to an eastward acceleration relative to the earth, and a decrease in the rate of rotation corresponds to a westward acceleration relative to the earth. In a rotating reference frame, these effects appear as a Coriolis force.

Characteristics of the Coriolis force:

The Coriolis force is proportional to the velocity of the fluid parcel relative to the Earth, and is zero when the fluid parcel has zero velocity in the rotating reference frame. The magnitude of the Coriolis force also increases with latitude and is zero at the equator, even if the fluid parcel has non-zero velocity at the equator. The Coriolis force always acts in a direction that is perpendicular to the direction of the velocity: toward the right in the northern hemisphere and toward the left in the southern hemisphere.

Geostrophic balance:

Geostrophic balance occurs when the pressure gradient force acting on a fluid parcel exactly balances the Coriolis force acting in the opposite direction. In this case, the parcel experiences no net acceleration and has a velocity that is perpendicular to the pressure gradient and the Coriolis force. Since the Coriolis force is proportional to the velocity of the fluid parcel, a stronger pressure gradient requires that the fluid parcel have a larger velocity in order for geostrophic balance to occur. Since the Coriolis force increases with latitude, a larger velocity is required for geostrophic balance at low latitudes than at high latitudes for the same pressure gradient force.

An example of geostrophic balance:

Variations in the surface energy fluxes cause the ocean to gain heat at low latitudes and lose heat at high latitudes. Ignoring heat transport, the heat gain causes the ocean column to expand at low latitudes and the heat loss causes the ocean column to contract at high latitudes. This produces a slight slope in the ocean surface (relative to the local horizontal) that is downward toward the pole, corresponding to a pressure gradient force towards the pole. Imagine an ocean initially at rest with respect to the earth. If the earth were not rotating, the pressure gradient force would move fluid toward the pole and thus flatten out the ocean surface. On a rotating earth, fluid parcels moving toward the pole conserve angular momentum and thus experience an eastward acceleration (appearing as a Coriolis acceleration in the rotating reference frame). This eastward acceleration causes the eastward component of the velocity of the parcels to increase. Parcels with an eastward velocity relative to the earth are rotating faster than the earth in the absolute reference frame and thus experience a centrifugal acceleration toward the equator (appearing as a Coriolis acceleration in the rotating reference frame). When the fluid parcels gain sufficient eastward velocity, the equatorward centrifugal force will exactly balance the poleward pressure gradient force in the absolute reference frame. This corresponds to a balance between an equatorward Coriolis force and a poleward pressure gradient force in the rotating reference frame. The fluid parcels will no longer have any component of velocity in the poleward direction since this would produce an eastward acceleration that would change the constant eastward velocity needed to maintain geostrophic balance. The slope in the sea surface remains the same since the poleward pressure gradient force is now balanced by the equatorward centrifugal force (absolute reference frame) or equatorward Coriolis force (rotating reference frame). This mechanism is a simplified explanation why sea surface height decreases from low latitudes to high latitudes and why surface ocean velocity is toward the east at middle latitudes.

Ekman transport:

Ekman transport occurs when the surface wind stress acting on the bulk Ekman layer (i.e., integrated over the layer of the ocean where downward momentum flux occurs) is balanced by the Coriolis force acting in the opposite direction. In this case, the bulk Ekman layer experiences no net acceleration and has a velocity (Ekman transport) that is perpendicular to the wind stress and the Coriolis force. Ekman transport is toward the right of the wind stress in the northern hemisphere and toward the left of the wind stress in the southern hemisphere. Ekman transport is proportional to wind stress.

An example of Ekman transport:

Consider an idealization of the ocean off the central California coast during summer. For simplicity assume that the coastline is constant in longitude, that the wind stress is purely in the equatorward direction, and that the ocean is initially at rest with respect to the earth. The wind stress will initially give the bulk Ekman layer an equatorward velocity, and as it moves toward the equator and away from the axis of rotation, conservation of angular momentum in the absolute reference frame will cause a westward acceleration (appearing as a Coriolis force in the rotating reference frame). Fluid parcels with a westward velocity relative to the earth are rotating slower than the earth in the absolute reference frame and thus experience a net gravitational acceleration toward the pole (appearing as a Coriolis acceleration in the rotating reference frame). When the bulk Ekman layer gains sufficient westward velocity (westward Ekman transport), the poleward gravitational force will exactly balance the equatorward wind stress in the absolute reference frame. This corresponds to a balance between a poleward Coriolis force and an equatorward wind stress in the rotating reference frame. The bulk Ekman layer will no longer have any component of velocity in the equatorward direction since this would produce a westward acceleration that would change the constant westward velocity needed to balance the wind stress. Since westward Ekman transport moves water near the ocean surface away from the coast, mass continuity requires that water from below upwell to replace it.

Another example of Ekman transport:

Consider the ocean near the Equator underneath the trade winds. For simplicity, we will assume that the wind stress is strictly westward and that the ocean is initially at rest with respect to the earth. The wind stress will initially give the bulk Ekman layer a westward velocity, and since it is rotating more slowly than the earth in the absolute reference frame, it will experience a net gravitational acceleration toward the pole (appearing as a Coriolis acceleration in the rotating reference frame) unless it is exactly on the equator. Parcels moving poleward are going toward the axis of rotation, and conservation of angular momentum in the absolute reference frame will cause an eastward acceleration (appearing as a Coriolis force in the rotating reference frame). When the bulk Ekman layer gains sufficient poleward velocity (poleward Ekman transport), the eastward acceleration due to conservation of angular momentum will exactly balance the westward wind stress. This corresponds to a balance between an eastward Coriolis force and a westward wind stress in the rotating reference frame. The bulk Ekman layer will no longer have any component of velocity in the westward direction since this would produce a poleward acceleration that would change the constant poleward velocity needed to balance the wind stress. Another way to look at it is that the westward wind stress applies the exact torque needed to change angular momentum such that the bulk Ekman layer always rotates at the same rate as the Earth as it moves poleward. Since poleward Ekman transport moves water near the ocean surface away from the equator, mass continuity requires that water from below upwell to replace it.

Ekman transport at middle latitudes:

Eastward wind stress occurs at middle latitudes and produces equatorward Ekman transport. Maximum values of wind stress and transport occur around 40-50° latitude.

Ekman pumping:

Since westward trade winds at low latitudes produce poleward Ekman transport and eastward winds at middle latitudes produce equatorward Ekman transport, there is convergence within the Ekman layer of the subtropical gyre. Mass continuity requires downward Ekman pumping at the base of the Ekman layer where horizontal convergence occurs. Since equatorward Ekman transport is maximum around 40-50° latitude and becomes weaker at latitudes closer to the pole, divergence occurs within the Ekman layer of the subpolar gyre, which leads to upward Ekman pumping. Upward Ekman pumping occurs if the curl of the wind stress is positive in the northern hemisphere or negative in the southern hemisphere, and downward Ekman pumping occurs if the curl is negative in the northern hemisphere or positive in the southern hemisphere.

Sverdrup transport:

Downward Ekman pumping in the subtropical gyre requires horizontal divergence within the deep ocean for mass continuity. In steady state conditions, this divergence cannot occur in the zonal (east-west) direction due to the finite size of ocean basins. If divergence occurs by movement in the equatorward direction, the increase in distance from the axis of rotation will result in a westward acceleration due to conservation of angular momentum in the absolute reference frame (appearing as a Coriolis acceleration in the rotating reference frame). It is essential to keep in mind, however, that the strength of the Coriolis force decreases with latitude even if the velocity remains constant. This means that fluid parcels experience less Coriolis turning the closer they are to the equator and thus retain a larger equatorward component of their velocity. The increase in the equatorward component of velocity with decreasing latitude produces the divergence in the deep ocean that is necessary to be physically consistent with downward Ekman pumping. The westward Coriolis acceleration will cause ocean mass to “pile up” on the western side of the ocean basin, thus causing a downward slope of the ocean surface toward the east. The resulting eastward pressure gradient force opposes the westward Coriolis force so that the equatorward Sverdrup transport is in geostrophic balance. Mass continuity requires a return flow toward the pole, which occurs in a narrow western boundary current.