# SIO 117: The Physical Climate System

Spring 2007

Homework #6

Due May 31 in class

1. This problem is a modification of Hartmann 7.7.

   Suppose that an ocean basin experiences wind stress $\tau_x = A \sin(\pi\, y / L)$ and $\tau_y = 0$. Only consider the domain $-L < y < L$. Approximate the Coriolis parameter as

   $f = f_0 + \beta_0\, y$, where $f_0$ and $\beta_0$ are constants.

   a) Derive equations for the vertically averaged Ekman velocities $\langle u_E \rangle$ and $\langle v_E \rangle$ in the zonal and meridional directions (units m $s^{-1}$). Let $h_E$ be the depth of the Ekman layer. For simplicity, assume that $\beta_0 = 0$.

   b) Derive an equation for the vertical velocity $w_E$ at the bottom of the Ekman layer. For simplicity, assume that $\beta_0 = 0$ and that $h_E$ is constant.

   c) Derive an equation for the vertically averaged velocity $\langle v_D \rangle$ in the deep ocean (below the Ekman layer). Let $h_D$ be the depth of the ocean below the Ekman layer. Assume that $\beta_0 = 0$ except in relation to $\langle v_D \rangle$ as in Eq. 7.12.

   d) Plot qualitatively how $\tau_x$, $\langle v_E \rangle$, $w_E$, and $\langle v_D \rangle$ vary on the interval $-L < y < L$. Draw another sketch in the *x*-*y* plane with arrows indicating the directions of $\tau_x$, $\langle v_E \rangle$, and $\langle v_D \rangle$ at values of $y = -L / 2$, $y = 0$, and $y = +L / 2$ (do not draw an arrow if the value is zero). Write "up", "down", or "zero" to indicate whether $w_E$ is upward, downward, or zero at $y = -L / 2$, $y = 0$, and $y = +L / 2$.

2. Using your answers to the first problem, calculate numerical values for $h_E = 100$ m, $h_D = 4000$ m, $L = 2000$ km, and $\rho_o = 1025$ kg $m^{-3}$, and $A = 0.1$ N $m^{-2}$ (corresponding to a near-surface wind speed of about 9 m $s^{-1}$). Let $y = 0$ at 30°N.

   a) Calculate the value of $f_0$ for 30°N.

   b) Calculate the value of $\beta_0$ for 30°N. Recall that $\beta = df / dy = a^{-1}\, df / d\phi$, where $a$ is the radius of the earth and $\phi$ is latitude.

   c) Calculate $\langle v_E \rangle$ at $y = 0$, y = –1000 km, and y = +1000 km (units cm $s^{-1}$).

   d) Calculate $w_E$ at $y = 0$, y = –1000 km, and y = +1000 km (units cm $day^{-1}$).

   e) Calculate $\langle v_D \rangle$ at $y = 0$, y = –1000 km, and y = +1000 km (units cm $s^{-1}$).

   f) If the longitudinal width of the basin is 4000 km, what is the total amount of water transported by the ocean below the Ekman layer at the latitude of 30°N? (units Sverdrup = $10^6$ $m^3$ $s^{-1}$). How does this value compare to the total transport by the Gulf Stream reported at the bottom of p. 200 of Hartmann?