

Notes on Wind Stress Forcing of the Ocean

Vorticity Equation

The horizontal momentum equations may be converted into an equation describing the vertical component of absolute vorticity η ,

$$\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y} = - \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \eta + \dots$$

where u is the wind in the zonal (x) direction and v is the wind in the meridional (y) direction. The left side shows the local change in time and rate of change due to advection. The right side shows the rate of change due to horizontal divergence with the “tilting” and “solenoidal” terms left off since they are small for large-scale flows.

Absolute vorticity is the sum of f , the Coriolis parameter (Earth’s rotation), and ζ , the relative vorticity (local rotation) of the flow

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

Assuming equilibrium, the local change in time is zero and horizontal advection of absolute vorticity balances divergence of absolute vorticity. Since for large-scale midlatitude flows ζ is much less than f , we can reduce the vorticity equation to

$$u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} \approx - \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) f.$$

Recalling that f only varies with latitude (y) and using the continuity equation for an incompressible fluid

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

we obtain

$$v \frac{\partial f}{\partial y} \approx f \frac{\partial w}{\partial z}.$$

Replacing with $\partial f / \partial y$ with the conventional symbol β yields the first equation listed in Deser et al. (1999).

Convergence in the Ekman layer (the near-surface part of the ocean affected by wind stress) will result in a profile where the vertical motion w is downward. Vertical motion is strongest near the surface and goes to zero at depth, so $\partial w / \partial z < 0$. Since $\partial f / \partial y$ is always positive, the meridional velocity v must have the opposite sign of f (i.e., negative and southward in NH and positive and northward in SH) and thus is always equatorward. Divergence in the Ekman layer will result in positive $\partial w / \partial z$, so meridional velocity must have the same sign as f (i.e., northward in the NH and southward in the SH) and thus is always poleward.

Wind Stress Curl

The stress force of wind gives the ocean an initial velocity in the direction of the wind, but the Coriolis effect due to the Earth's rotation exerts an acceleration proportional to velocity and at right angles to the direction of motion. This turns the ocean velocity away from the direction of the wind, and the resulting Ekman transport is 90° to the right of the wind direction in the NH and 90° to the left of the wind direction in the SH. The Ekman transport may be calculated by balancing the wind stress force with the Coriolis force due to ocean velocity integrated over the Ekman layer,

$$\tau_x = -\rho_o f V_E, \quad \tau_y = \rho_o f U_E$$

where ρ_o is the density of seawater, τ_x and τ_y are the wind stresses in zonal and meridional directions, and

$$U_E = \int_{z_E}^0 u dz, \quad V_E = \int_{z_E}^0 v dz,$$

where z_E is the base of the Ekman layer.

Using the continuity equation again we can calculate the vertical motion through the base of the Ekman layer, w_E , resulting from layer mean Ekman transports

$$-\int_{z_E}^0 \frac{\partial w}{\partial z} dz = \frac{\partial U_E}{\partial x} + \frac{\partial V_E}{\partial y}$$
$$w_E = \frac{\partial U_E}{\partial x} + \frac{\partial V_E}{\partial y}.$$

Replacing the Ekman transport terms with the wind stress terms yields the relationship between Ekman pumping and the curl of the wind stress

$$w_E = \frac{\partial}{\partial x} \frac{\tau_y}{\rho_o f} - \frac{\partial}{\partial y} \frac{\tau_x}{\rho_o f} = \mathbf{k} \cdot \nabla \times \frac{\boldsymbol{\tau}}{\rho_o f}.$$

Positive wind stress curl causes divergence in the Ekman layer and upward Ekman pumping and negative wind stress curl causes convergence in the Ekman layer and downward Ekman pumping.

The mean wind at midlatitudes is primarily zonal and from the west. This generates equatorward Ekman transport proportional to the magnitude of the wind. The magnitude of equatorward Ekman transport is maximum at the latitude where the wind is maximum and decreases towards both the pole and equator. This causes convergence in the Ekman layer and downward Ekman pumping in the subtropical gyre and divergence in the Ekman layer and upward Ekman pumping in the subpolar gyre. The easterly trade winds generate poleward Ekman transport, which also causes convergence in the subtropical gyre.

The previous equation

$$\beta v = f \frac{\partial w}{\partial z}$$

can be integrated from the level of zero vertical motion at depth in the ocean (z_o) to the base of the Ekman layer to derive the equation for Sverdrup balance,

$$\beta \int_{z_o}^{z_E} v dz = f \int_{z_o}^{z_E} \frac{\partial w}{\partial z} dz = f w_E$$

$$\beta V_g = \frac{f}{\rho_o} \mathbf{k} \cdot \nabla \times \frac{\boldsymbol{\tau}}{f},$$

where V_g is the vertically integrated meridional geostrophic transport. This is equation (1) in Deser et al. (1999). Rearranging the curl term to separate $\boldsymbol{\tau}$ and f yields

$$\frac{f}{\rho_o} \mathbf{k} \cdot \nabla \times \frac{\boldsymbol{\tau}}{f} = \frac{f}{\rho_o} \left[\frac{\partial}{\partial x} \left(\frac{\tau_y}{f} \right) - \frac{\partial}{\partial y} \left(\frac{\tau_x}{f} \right) \right] = \frac{f}{\rho_o} \left[\frac{1}{f} \frac{\partial \tau_y}{\partial x} - \frac{1}{f} \frac{\partial \tau_x}{\partial y} + \frac{\tau_x}{f^2} \frac{\partial f}{\partial y} \right] = \frac{1}{\rho_o} \mathbf{k} \cdot \nabla \times \boldsymbol{\tau} + \frac{\tau_x \beta}{\rho_o f}$$

Note that the last term is $-\beta$ multiplied by the vertically integrated Ekman transport, V_E (as mentioned in Deser et al. 1999). Thus,

$$\beta V_g = \frac{1}{\rho_o} \mathbf{k} \cdot \nabla \times \boldsymbol{\tau} - \beta V_E$$

$$\beta (V_g + V_E) = \frac{1}{\rho_o} \mathbf{k} \cdot \nabla \times \boldsymbol{\tau},$$

where the right side of the bottom equation is the Sverdrup transport mentioned in Deser et al. (1999). The left side of the bottom equation shows that the Sverdrup transport is the total meridional transport of β , the sum of the transport by the Ekman layer (V_E) and the layer below the Ekman layer (V_g). I suppose V_g is called the geostrophic transport because there is no contribution by friction in the lower layer (unlike the Ekman layer) and steady flow must hence be in geostrophic balance.

Positive wind stress curl causes poleward Sverdrup transport and negative wind stress curl causes equatorward Sverdrup transport. Return flow occurs along the western boundary and is poleward in the subtropical gyre and equatorward in the subpolar gyre.

Physical Interpretation

To interpret the equations, I like to understand the fundamental physical processes involved. Since it is more difficult for me to conceptualize pseudoforces like the Coriolis effect, I often consider flow in a non-rotating coordinate system.

Consider an ocean basin that is stationary with respect to the Earth but has angular velocity from the Earth's rotation. If a westerly wind stress is applied, the angular velocity of the ocean increases and the centrifugal acceleration no longer balances the gravitational acceleration. This causes outward radial motion which gravity constrains to be towards the equator (Ekman transport). If the zonal wind stress has a maximum, the equatorward motion converges equatorward and diverges poleward of the wind maximum. By mass continuity there must be upward motion under the divergence and downward motion under the convergence (Ekman pumping). Again by mass continuity there must be underlying convergence with the upward motion and underlying divergence with the downward motion. The subpolar region of underlying convergence brings together water from different latitudes and different angular velocities. Because there is less area at higher latitudes, the lower latitudes with greater angular velocity dominate and angular velocity increases. To conserve angular momentum, the radius of angular motion must decrease through poleward motion. The opposite happens for the subtropical region of underlying divergence.