SIO 217B Atmospheric and Climate Sciences II

Exercise B

1. a) Consider a tank of water rotating at angular speed Ω. Derive an expression for the height of the water surface, h, as a function of radius, r. Let h_0 be the height of the water surface at the center of the tank.

   b) Assuming that the tank has a radius of 2 m and rotates once every 10 seconds, plot h as a function of r. Let h_0 be 20 cm.

2. a) The Earth has an oblate shape due to centrifugal acceleration from rotation. A simple way to find the variation of radius with latitude is to calculate the height of a constant geopotential surface. Recall from basic physics that acceleration is related to mass-normalized potential as follows.

   $$d\mathbf{U}/dt = -\nabla\Phi$$

   Thus gravity geopotential Φ_g can be derived from gravity acceleration $\boldsymbol{g}$.

   $$\boldsymbol{g} = -G\, M_E\, \hat{\mathbf{r}} / r^2 \qquad\qquad \Phi_g = -G\, M_E / r$$

   Derive equations for centrifugal acceleration $d\mathbf{U}_{cent}/dt$ and centrifugal potential Φ_{cent} as a function of latitude ϕ, rotation rate Ω, and radius of the Earth a.

   b) Let a_p be the radius of the Earth at the poles, and let h be the difference between radius r of the oblate surface and that of a perfect sphere with radius a_p.

   $$h(\phi) = r(\phi) - a_p$$

   At equilibrium, the surface of a rotating planet will have a constant value of combined gravity and centrifugal geopotential for all latitudes. Use this principle to derive an equation for h in terms of Ω, a_p, ϕ, and g, the value of gravitational acceleration at the poles. For simplicity, use the gravity potential equation for a perfect sphere and drop second-order terms assuming that $h \ll a_p$.

   c) Plot h as a function of latitude from the North Pole to the Equator. You may use values of global average radius and gravitational acceleration for a_p and g.

   It turns out that the difference between Earth's polar radius and equatorial radius is actually 21 km, not 11 km. The reason for this is that Earth's oblate shape means the gravity potential does not perfectly correspond to that for a sphere.